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Efficient Scheme for One-Way Quantum Computing in Thermal Cavities

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Abstract We propose a practical scheme for one-way quantum computing based on efficient generation of 2D cluster state in thermal cavities. We achieve a controlled-phase gate that is neither sensitive to cavity decay nor to thermal field by adding a strong classical field to the two-level atoms. We show that a 2D cluster state can be generated directly by making every two atoms collide in an array of cavities, with numerically calculated parameters and appropriate operation sequence that can be easily achieved in practical Cavity QED experiments. Based on a generated cluster state in $\mathrm{Box}^{(4)}$ configuration, we then implement Grover's search algorithm for four database elements in a very simple way as an example of one-way quantum computing.

Keywords Cluster state · Quantum computation · Cavity QED

Over the past few years, the construction of a practical quantum computer has become a challenging goal for experimentalists. It is well known that the building blocks of a general quantum computer are single-qubit rotations and two-qubit quantum gates [1]. Recently, Briegel and Raussendorf [2, 3] proposed a new idea for constructing quantum computer, known as one-way quantum computing, which shows that preparation of a particular entangled state, called cluster state, accompanied with local single qubit measurements, is sufficient for simulating any arbitrary quantum logic operations. Cluster state as a universal resource for general quantum computing has drawn extensive research interests [4–8]. Moreover, one way quantum computing by optical elements based on four-qubit cluster states was recently demonstrated experimentally [9]. It is hoped that experimental difficulties in performing complex quantum gates may be overcome by one-way quantum computing based on the generation of cluster state.

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A cluster state $|\psi\rangle_c$ can be visualized as a collection of qubits positioned at certain sites of a 2D lattice structure with lines connecting them, which can be specified by the following set of eigenvalue equations:

$$K^{(a)}|\psi\rangle_c = (-1)^{\kappa_a}|\psi\rangle_c \tag{1}$$

with the correlation operators

$$K^{(a)} = \sigma_x^{(a)} \bigotimes_{b\in nghb(a)} \sigma_z^{(b)} \tag{2}$$

where $nghb(a)$ is set of all the neighbors of any site a of the lattice, and $\kappa_a \in \{0, 1\}$. To generate an arbitrary cluster state, one can first initialize each qubit in state $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, where $|0\rangle$ and $|1\rangle$ are the computational basis, and then perform controlled-phase operations between all neighboring qubits connected by the lines of the lattice.

Cavity QED system is known to be a qualified candidate for quantum information processing [10]. However, up to now, one-way quantum computing based on Cavity QED techniques has neither been proposed theoretically nor been carried out experimentally. The main difficulty lies in generating an arbitrary 2D cluster state. Proposed schemes of generating cluster states using Cavity QED methods [11–14] are difficult for practical and scalable experiments either due to the decoherence of the cavity field mode or due to the sensitivity of thermal field. Besides, most of the schemes are mainly focused on linear cluster state prepared in one dimension, which are not suitable for use as substrate for quantum computation since one-way quantum computing based on 1D cluster state can be efficiently simulated by classical computer [15, 16] and most proposals for generating 2D cluster state are inefficient, as they first need to generate several 1D cluster states and then collide them into a 2D configuration.

In the present work, we propose a practical scheme for one-way quantum computing based on efficient generation of 2D cluster state in thermal cavities. Compared to [11–14], our scheme is neither sensitive to cavity decay nor to thermal field, since the evolution of the atomic states is independent of the cavity field mode, which is achieved by adding a strong classical field to our system. In addition, our implementation of controlled-phase gate does not need any auxiliary state, i.e. two-level atoms are used instead of three-level atoms, which further reduces experimental difficulties. On the other hand, the 2D cluster state is generated in a direct and efficient manner in our scheme by appropriately choosing the initial velocity of each atoms as well as the time delay between atom preparations and placing an array of cavities at certain locations in the path atoms passing through, so that every two atoms can collide in a certain cavity and be subjected to entanglement generation as in [17]. We give the generation of arbitrary 4-qubit cluster state as an example with reasonable parameters and concrete operation sequence, and show that our scheme can perform one-way quantum computing process such as Grover's search algorithm for four database elements in a simple and convenient way that is within the current experimental techniques.

Our generation of entanglement in cluster state is based on the interaction between two identical two-level atoms and a single-mode cavity field driven by a classical field. In the rotating wave approximation, the Hamiltonian for such a system is given by (assuming $\hbar = 1$) [18]

$$H = \sum_{j=1}^{2} \omega_0 \sigma_{z,j} + \omega_a a^\dagger a + \frac{1}{2}\sum_{j=1}^{2}\left[g(a^\dagger \sigma_j^- + a\sigma_j^+) + \Omega(\sigma_j^+ e^{-i\omega t} + \sigma_j^- e^{i\omega t})\right] \tag{3}$$

where a and $a^\dagger$ are the annihilation and creating operators for cavity mode, $\sigma_{z,j} = |e\rangle_{jj}\langle e| - |g\rangle_{jj}\langle g|$, $\sigma_j^+ = |e\rangle_{jj}\langle g|$, $\sigma_j^- = |g\rangle_{jj}\langle e|$, with $|e\rangle_j(|g\rangle_j)$ being the excited (ground) state of the jth atom. ω_0, ω_a and ω are the frequencies for atomic transition, cavity mode, and classical field respectively. g is the atom-cavity coupling strength and Ω is the Rabi frequency of the classical field. Assume that $\omega_0 = \omega$. Then we can obtain the following interaction Hamiltonian in the interaction picture:

$$H_i = \sum_{j=1}^{2}\left[\frac{\Omega}{2}(\sigma_j^+ + \sigma_j^-) + \frac{g}{2}(e^{-i\delta t}a^\dagger\sigma_j^- + e^{i\delta t}a\sigma_j^+)\right] \tag{4}$$

with $\delta = \omega_0 - \omega_a$. For the new atomic basis $|\pm\rangle_j = (|g\rangle_j \pm |e\rangle_j)/\sqrt{2}$, then we make a further transformation with rotation with respect to the terms regarding Ω in (4), and obtain

$$H_I = \frac{g}{4}\sum_{j=1}^{2}\big(|+\rangle_{jj}\langle +| - |-\rangle_{jj}\langle -| + |+\rangle_{jj}\langle -|e^{i\Omega t} - |-\rangle_{jj}\langle +|e^{-i\Omega t}\big)e^{-i\delta t}a^\dagger + H.c. \tag{5}$$

Free Hamiltonian $H_0 = \frac{\Omega}{2}\sum_{j=1}^{2}(|+\rangle_{jj}\langle +| - |-\rangle_{jj}\langle -|)$ has been used here for the transformation. Assuming that $\Omega \gg \delta, g$, we can neglect the fast oscillating terms. Then obtain the effective interaction Hamiltonian,

$$H_e = \frac{g}{2}(e^{-i\delta t}a^\dagger + e^{i\delta t}a)\sigma_x \tag{6}$$

where $\sigma_x = \frac{1}{2}\sum_{j=1}^{2}(\sigma_j^+ + \sigma_j^-)$. The evolution operator for Hamiltonian (6), which was first proposed for trapped-ion system [19], can be written as

$$U_e(t) = e^{-iA(t)\sigma_x^2}e^{-iB(t)\sigma_x a}e^{-iC(t)\sigma_x a^\dagger} \tag{7}$$

By solving the Schrödinger equation $idU_e(t)/dt = H_iU_e(t)$, we can obtain $B(t) = g(e^{i\delta t} - 1)/2i\delta$, $C(t) = -g(e^{-i\delta t} - 1)/2i\delta$, $A(t) = g^2[t + (e^{-i\delta t} - 1)/i\delta]/4\delta$. Choosing $\delta t = 2\pi$, we have $B(t) = C(t) = 0$. Thus we get the evolution operator of the system,

$$U(t) = e^{-iH_0t}U_e(t) = e^{-i\Omega t\sigma_x - i\lambda t\sigma_x^2} \tag{8}$$

where $\lambda = g^2/4\delta$. If we first apply single-qubit Hadamard gate on both atoms, and then set the interacting time t and Rabi frequency Ω appropriately so that $\lambda t = \pi/2$ and $\Omega t = (2k + \frac{1}{2})\pi$ (k is integer), followed by implementing again the Hadamard gate on both atoms, then we obtain a controlled quantum phase gate with computational basis $|0\rangle, |1\rangle$ represented by $|e\rangle, |g\rangle$:

$$\begin{cases} H^{\otimes 2}U(t)H^{\otimes 2}|g\rangle_1|g\rangle_2 = -|g\rangle_1|g\rangle_2 \\ H^{\otimes 2}U(t)H^{\otimes 2}|g\rangle_1|e\rangle_2 = |g\rangle_1|e\rangle_2 \\ H^{\otimes 2}U(t)H^{\otimes 2}|e\rangle_1|g\rangle_2 = |e\rangle_1|g\rangle_2 \\ H^{\otimes 2}U(t)H^{\otimes 2}|e\rangle_1|e\rangle_2 = |e\rangle_1|e\rangle_2 \end{cases} \tag{9}$$

In order to generate an arbitrary two-dimensional cluster state using the controlled quantum phase gate given in (9), we first assume that the horizontal velocity v_i and time t_i of the ith atom emitting from the single-atom source has been pre-selected according to our

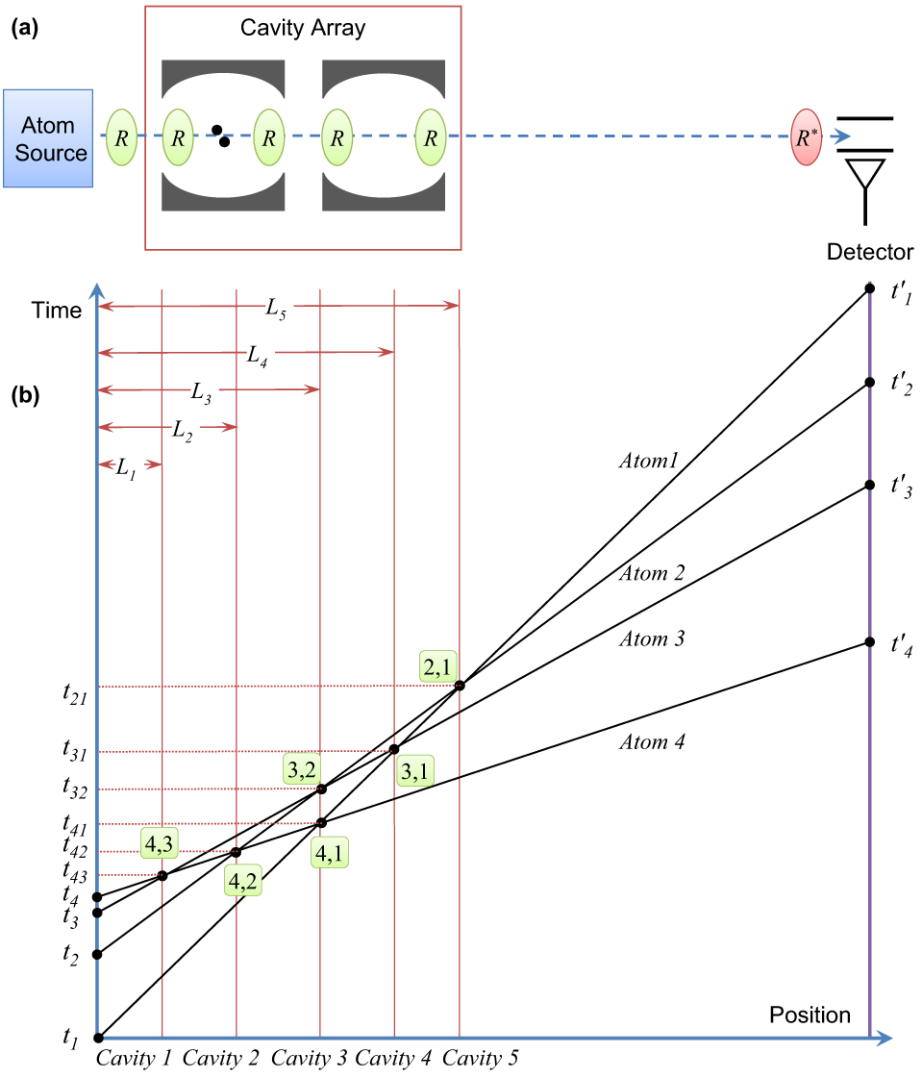


Fig. 1 **(a)** Proposed experimental set-up for one-way quantum computing in an array of thermal cavities. R represents for Ramsey zone with $\pi/2$ resonant pulse added at the time of atom collisions. R* denotes for Ramsey zone with detuned $\pi/2$ pulse. **(b)** Space-time diagram for the sequence of events. Every two atoms are made to cross the center of certain cavity simultaneously before finally reaching the detector

need, and the vertical position of each atom are slightly different. After the atoms initially in ground state move out horizontally from the source, a $\pi/2$ classical resonant pulse R is added to each atom so that they are prepared in the state $(|0\rangle + |1\rangle)/\sqrt{2}$. Our next aim is to let every two atoms collide in a certain cavity and so that they may undergo the evolution in (8), and an arbitrary 2D cluster state can thus be formed. For N atoms forming a N-qubit cluster state, calculations show that this requires us to place $(2N - 3)$ cavities in a array as depicted in Fig. 1(a). Suppose the kth cavity center is at a distance L_k from the single-atom source, the following equations should be satisfied in order to achieve collisions between

Table 1 One numerical solution for (10) in the case of $N = 4$. Part of the quantities are set as constants and others are variables being solved

v_1	v_2	v_3	v_4	
100 m/s	122 m/s	146 m/s	250 m/s	
t_1	t_2	t_3	t_4	
0 ms	0.359 ms	0.471 ms	0.500 ms	
L_1	L_2	L_3	L_4	L_5
1.00 cm	3.35 cm	8.33 cm	15.00 cm	20.00 cm

every two atoms.

$$L_{i+j-2}\left(\frac{1}{v_j}-\frac{1}{v_i}\right)=t_i-t_j \tag{10}$$

where $i = 1, 2, \ldots, N-1$; $j = i+1, i+2, \ldots, N$ and we set $t_1 = 0$ for simplicity. Equation (10) contains $N(N-1)/2$ nonlinear equations and $4(N-1)$ variables. Calculations show that it has a group of solutions for $N \leq 6$ and may not give any solution for $N \geq 7$. For pedagogical reasons, we are aimed at the relatively simple but important case of $N = 4$, where one can get many reasonable solutions that satisfy (10). Figure 1(b) shows a possible case qualitatively. The analytical solution we obtained is rather complicated, but our numerical calculation gives many quantitative solutions that are appropriate in practical experiments. Table 1 shows one example.

Since we only want the atom pairs representing neighboring qubits in the cluster state to undergo the controlled-phase operation in (8), in all the other case, the cavity is set far off resonance by a large electric field applied across the cavity mirrors. This field stark-shifts the atomic levels far off resonance, so that the atom-cavity interaction is then negligible [20]. The interaction time in (8) is also controlled strictly in the same way together with control of the length of the strong classical pulse added. In fact, small difference of the interaction time between two atoms will not cause measurable errors [18, 21].

Now we'd like to show how to implement one-way computing based on the generation of cluster state in our scheme. Here we use the Grover's search algorithm for four unsorted database elements as an example, the circuit model [22] and one-way computing model [9] of which have been shown in Fig. 2. The cluster state we need to generate is in Box$^{(4)}$ configuration ($|\psi\rangle_c = |0\rangle_1|+\rangle_2|0\rangle_3|+\rangle_4 + |0\rangle_1|-\rangle_2|1\rangle_3|-\rangle_4 + |1\rangle_1|-\rangle_2|0\rangle_3|+\rangle_4 + |1\rangle_1|+\rangle_2|1\rangle_3|-\rangle_4$), which entails us to tune the cavity field to near resonance when atom pairs (4,3), (4,1), (3,2), (2,1) collide in Cavity 1, 3 and 5 at time $t_{43}, t_{41}, t_{32}, t_{21}$ as in Fig. 1(b), while $\{v_1, v_2, v_3, v_4\}$, $\{t_1, t_2, t_3, t_4\}$ and $\{L_1, L_2, L_3, L_4, L_5\}$ are pre-set to values satisfying (10), such as in Table 1.

After t_{21}, the cluster state has been generated and we are then to perform single-qubit measurements in the order of atom 4-3-2-1. Atom 4 and atom 3 should be measured in the basis $(|0\rangle \pm e^{i\alpha}|1\rangle)/\sqrt{2}$ and $(|0\rangle \pm e^{i\beta}|1\rangle)/\sqrt{2}$, which can be achieved simply by adding $\pi/2$ Ramsey pulse with frequency ω_r satisfying $(\omega_r - \omega_0)T = \alpha(\beta)$ respectively, (ω_0 is the atomic transition frequency and T is the coherent interaction time between the pulse and atom) and then measuring the atom in basis $\{|0\rangle, |1\rangle\}$ through Field Ionization Detector. Atom 2 and Atom 1 carry the read-out qubits in the cluster state model and should go through a σ_z and a Hadamard gate, which can be realized by adding the same $\pi/2$ Ramsey pulse with frequency ω_r satisfying $(\omega_r - \omega_0)T = \pi$, before final measurements by the Detector in computational basis. If the measurement results of the four flying atom at time sequence t_4', t_4', t_2', t_1' are denoted by r_4, r_3, r_2, r_1, then the one-way quantum computing process

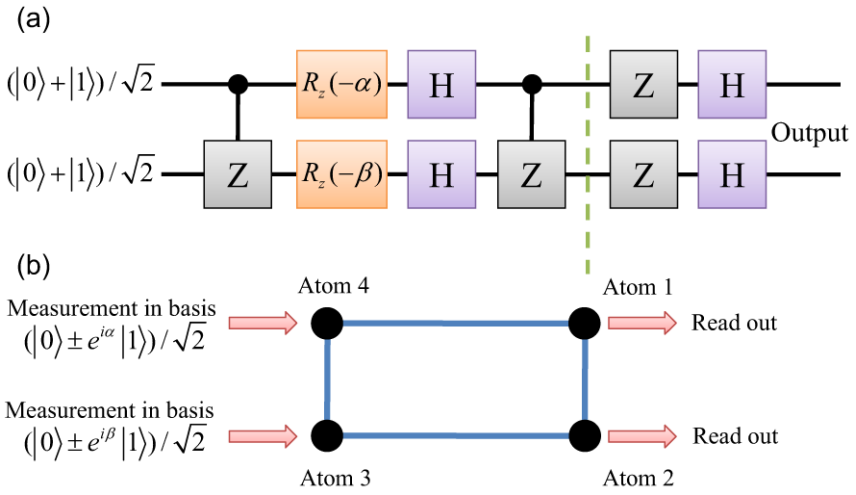
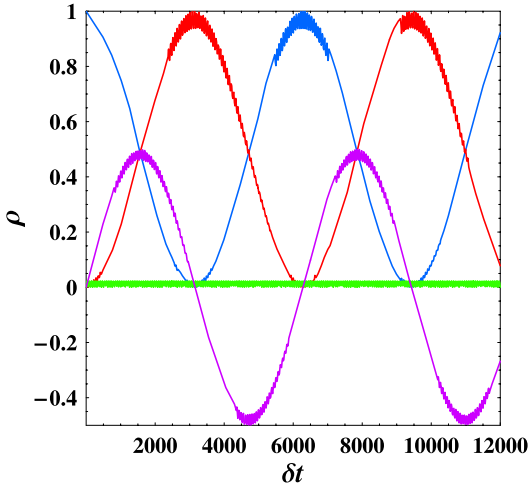


Fig. 2 **(a)** Circuit model of Grover's search algorithm for four database elements. The two encoded qubits pass through a series of unitary quantum logic gates before being measured in computational basis as output of the search algorithm. α, β are determined by the "Oracle", and setting $\alpha\beta$ to $\pi\pi$,π0,0π,00 corresponds to the marked element encoded as 00,01,10,11. **(b)** Cluster state quantum computing model for the same algorithm. The physical qubits carried by Atom 4,3 are measured in any order, changing the states of Atom 2,1 to states of the encoded qubits in the circuit model at the same stage

Fig. 3 Coherent evolution of two atoms in a thermal cavity according to (8), with the parameters $\delta = g$, $\Omega = 5\delta$ and $n_{\text{th}} = 1$. The two atoms are assumed to be initially in the ground states $|g\rangle_1|g\rangle_2$. The *red*, *blue*, *green* and *purple curves* represent for $\rho_{gg,gg}$, $\rho_{ee,ee}$, $\text{Re}(\rho_{gg,ee})$ and $\text{Im}(\rho_{ee,ee})$ respectively. The small deviation from ideal situation on the curves are small oscillations with a high frequency



succeeds in giving us the search result encoded as $\{r_1 \oplus r_3, r_2 \oplus r_4\}$ if r_3 and r_4 are not both zero [9].

We now discuss further the experimental feasibility of our one-way quantum computing scheme. From (8), we note that the photon-number-dependent parts in the evolution operator are canceled with the strong resonant classical field added, thus our scheme can be done in thermal cavities and is also insensitive to cavity decay, greatly reducing experimental difficulties. The robustness of our scheme to thermal field is illustrated in Fig. 3, where we simulate the time evolution curves of the two atoms in the cavity by solving an

appropriate master equation and allowing for heating in form of quantum jumps described by jump operators $\sqrt{\Gamma n_{\rm th}}a$ and $\sqrt{\Gamma(n_{\rm th}+1)}a^{\dagger}$ (Γ and $n_{\rm th}$ are the typical heating rate and the mean excitation of the photon). Figure 3 clearly shows that we have a coherent evolution of the atomic state which is not entangled with the cavity. Besides, in obtaining (8), there is no requirement that the atom-cavity detuning should be much larger than the atom-cavity coupling strength. Operation time can be thus shortened, which is also important in view of decoherence.

Moreover, the velocities of the atoms can be selected by Doppler-selective optical pumping techniques, with a precision of ± 2 m/s, and the timing of each atom preparation can be controlled to a precision of 2 μs [10]. Besides, the Cavity QED experimental apparatus are typically 20 cm in length [10]. These validates our choice of v_i, t_i and L_i. It is also worth mentioning that since the waist of the cavity field is at most a few mm, which is much smaller than the distance between the cavities, it is unlikely that our space-time arrangement of atoms will cause more than two atoms simultaneously crossing the center of the cavity field.

The radiative time for Rydberg atoms with principal quantum numbers around 50 is $T_{\rm r} = 3 \times 10^{-2}$ s, and the coupling constant is $g = 2\pi \times 25$ kHz [10]. The corresponding photon storage time in a cavity can reach $T_{\rm c} = 1$ ms [10]. In the present scheme, the virtually excited cavities have only a small probability, about 1%, of being excited during the passage of the atom pairs through them. Thus the efficient decay time of the cavity $T_{\rm ceff} \sim 0.1$ s. Choosing $\delta = g$, direct calculation shows that the interaction time is on the order of 10^{-5} s, which is much shorter than the cavity decay time $T_{\rm ceff}$. The time needed for the whole one-way quantum computing process can be controlled within a few ms according to the choice of t_i, apparently smaller than the radiative time of the Rydberg atoms $T_{\rm r}$, rendering our scheme insensitive to the decoherence of atoms.

Errors in our scheme will mainly come from the process of entanglement generation in cavities and the single-qubit measurement process after generating the cluster state. In the former case, the errors can be induced by Start shift on the states $|+\rangle_j$ and $|-\rangle_j$ as we've discarded the fast-oscillating terms in obtaining (6), and by pulse imperfections and initial cavity Fock state. Ref. [23] shows that these errors will only have slight influence on the fidelity of the entangled state obtained. In the latter case, we note that the measurement efficiency of ionization detectors can be more than 80% with current techniques [24]. However, we're still expecting higher measurement efficiency in Cavity QED experiments.

For one-way computing of quantum algorithms that need no more than 6 physical qubits, our scheme does not need much change. However, for a large number of qubits, (10) cannot be satisfied by simply setting $\{v_i\}$, $\{t_i\}$, $\{L_i\}$ before the computing process. One of the possible ways to solve this problem is to change the velocity of the atoms in the midway. Although Rydberg atoms are neutral and usually not easy to accelerate/decelerate, we note that recent progress on Cavity QED experiment with optically transported atoms can manipulate the motion of atoms by resorting the optical dipole force [25, 26]. Such technique can provide a moderate acceleration for optically trapped atoms and deliver them deterministically, which might make our scheme still possible to work for relatively large scale of one-way quantum computing process.

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